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TIME-VARYING AND SCALE EFFECT OF PAYOFF UNCERTAINTY ON NASH EQUILIBRIUM PAYOFF IN 2 × 2 SIMULATION-BASED GAME: A WEIBULL DISTRIBUTION CASE

Abstract. *We characterize the time-varying effect from payoff uncertainty where the shape parameter of Weibull distribution is changed in 2 × 2 simulation-based game. We also analyze the scale effect of strategic payoffs on the Nash equilibrium payoffs in different time-varying situations. We show that the players choose the dominant strategy but have higher expected value of the Nash equilibrium payoff with a risk premium whatever the change of time-varying effect and scale effect are. We also show that the time-varying effect from early harvest to late harvest leads to a lower average and variance of the Nash equilibrium payoff. The scale effect induces in the higher average and variance of the Nash equilibrium payoff. If players want to earn higher Nash equilibrium payoff, then they need to compare the time-varying situation with different scale effect to know the specific risk range.*

Keywords: *Game theory, Decision-making, Weibull distribution, Payoff uncertainty.*

JEL Classification:C73, C88, D8

1. Introduction

Decision-making analysis is a method used to resolve decision problems depending on a specific phenomenon and to provide incentives to players. Game theory is a good tool for analyzing the decision-making process of players. To support realistic models, game theory questions regarding uncertainty have been investigated since the 1970s. For instance, Harsanyi (1973) modeled a mixed-strategy game model with payoff uncertainty characterized by an error following a uniform distribution.

Harsanyi showed the existence of a game equilibrium in the case where the strategic payoffs are slightly disturbed by an error. Cassidy, Field, and Kirby (1972) used discrete probability and a mixed strategy method to solve two-person and zero-sum games with random payoffs and to provide the concept for a satisfying criterion. Another method is based on the simulation-based game which considers the establishment of a game model, and the results of the game model are obtained by computer simulation (Vorobeychik and Wellman, 2008, Vorobeychik,2010). Lee (2014) determined the importance of the NE payoff when the game has a strategic payoff uncertainty. Lee and Lee (2014, 2015) indicated that the means between two strategies change the NE payoff distribution, which is not necessarily a normal distribution unless the mean difference of the two strategic payoffs is large enough. Previous literature has not addressed these questions, particularly regarding the optimal payoff in equilibrium and how players' payoffs change with varying time.

To address this issue, we wrote an intentionally simplified model with 2 players and 2 strategies, that is, a 2 × 2 game model. Players can consider the strategies as a dominant strategy (DS) and a non-dominant strategy (NDS). They also know the entire model structure, but do not know the realized strategic and equilibrium payoffs. The criterion of strategic payoff is that the DS payoff is larger than the NDS payoff regardless of the scale effect and time-varying effect of the DS payoffs. A distribution assumption is used as a proxy for uncertainty without considering the relationship between means and variances (Varian, 2009), moreover without the higher-order moments.

Here, we modeled a Weibull distribution to describe the payoff uncertainty and investigate the pattern of the NE payoffs according to a parametric change of the Weibull parameters, which include a scale parameter and a shape parameter.[1] Thus, the parameters of the Weibull distribution play an important role in the NE payoff. We used the scale parameter to describe the scale effect and shape parameter as a

1The Weibull distribution is always used in life data analysis and reliability engineering to measure failure rates and reliability according to the change in the shape parameter and the scale parameter. In the bathtub curve, the early-life failure is the observed failure of a product in the guarantee period where the condition that the shape parameter is less than 1 can be viewed as a short-term strategy where players can have an early harvest, whereas the wear-out failure can be viewed as a late harvest. See http://www.weibull.com/hotwire/issue14/relbasics14.htm time-varying effect. On the one hand, for a given shape parameter, the scale parameter can show how the shift of the DS payoffs leads to a change in the NE payoffs. The higher the scale parameter of the DS payoff, the larger the NE payoff and the lower the NE risk, regardless of which outcomes take place in the early-harvest or late-harvest situations. On the other hand, for a given scale parameter, the shape parameter has properties of early-life and wear-out failure in the bathtub curve.

The remainder of this paper is structured as follows. Section 2 describes the game structure and simulation method. Section 3 explains the simulation results with respect to the interactive expected values and variances of the NE payoffs. Section 4 concludes the paper.

2. Model and Simulation Procedures

We will start with a 2 × 2 game model to illustrate how a player's incentive to choose the optimal strategy changes when players have realized the payoff after decision-making. The payoff matrix is illustrated in Table 1. Since the game model is a symmetric game, we will only discuss the behavior of Player 1.

Table 1. The payoff matrix of normal form game with dominant strategy

		Player 2	
		L	R
Player 1	U	X_2, X_2	10, X_1
	D	X_1, 10	5, 5

The players choose their strategies simultaneously. The DS of each player is characterized by the value of strategic payoff.

Assumption 1.

The strategic payoffs, X_1 and X_2, are variables, and $X_2 > X_1$.

By assumption 1, Strategy U is the DS of Player 1 and Strategy L is the DS of Player 2. Once incurred, if X_1 and X_2 are certain constants, then the NE without payoff uncertainty is (U, L) where Players 1 and 2 earn X_2.

Once the strategic payoffs have been added with uncertainty when players are making their decision, they become common knowledge. After this, players still choose simultaneously which strategy produces the maximum payoff within their information set, with random variables X_1 and X_2, the condition $E(X_2) > E(X_1)$, the realized timing of X_1 and X_2, and the strategic payoff distribution.

Define X_1 and X_2 as independent and identically distributed (i.i.d.) variables with a Weibull distribution, $X \sim$ Weibull (α, β, γ) with the probability density function [2]

$$f_X(x) = \frac{\gamma}{\beta}\left(\frac{x-\alpha}{\beta}\right)^{\gamma-1} e^{-\left(\frac{x-\alpha}{\beta}\right)^{\gamma}}$$

where γ is the shape parameter, α is the location parameter, and β is the scale parameter. We will assume $\alpha = 0$ to discuss the 2-parameter Weibull distribution where the mean is $\beta\,\Gamma((\gamma + 1) / \gamma)$ and the variance is $\beta^2 (\Gamma((\gamma + 2) / \gamma) - \Gamma((\gamma + 1) / \gamma)^2)$. If $\alpha = 0$ and $\beta = 1$, this is the standard Weibull distribution.

For what follows, it is convenient to define the time-varying effect of strategies. Let the shape parameter of the Weibull distribution denote the time-varying effect because the bathtub curve indicates the relationship of the range of γ and failure rates. We can distinguish the time-varying property of a strategy from the range of γ so that a 2 × 2 game can be used to discuss the time-varying cases. Generally, in terms of failure rates, wear-out failures occur in the range $\gamma > 1$ when a failure rate increases with time. Furthermore, early-life failures are defined as being when the failure rate of the product decreases with time for $\gamma < 1$. Finally, random failures are when the failure rate increases with time for $\gamma = 1$. We applied the concept of failure rates to the realized payoff. The situation of wear-out failures is regarded as either the long-term or late-harvest strategy because the payoffs are realized in a relatively long time frame. The early-life failures are viewed as either the short-term strategy or early-harvest strategy because the payoffs are realized in a relatively short time frame. The random failures are viewed as either the mid-term strategy or

2 The description of Weibull distribution is referenced on http://www.itl.nist.gov/div898/handbook/eda/section3/eda3668.htm.

normal-harvest strategy because the payoffs are realized in a normal time frame. Thus, there are four cases to investigate regarding how the parameter changes the NE payoff distribution. They are as follows.

Case	Situation	Distribution of X1	Distribution of X2
Case 1	Late harvest of X1 and X2	Weibull (0, 1, 2)	Weibull (0, β_2, 2) $1.1 \leq \beta_2 \leq 7$
Case 2	Early-harvest of X1 and X2	Weibull (0, 1, 0.2)	Weibull (0, β_2, 0.2) $1.1 \leq \beta_2 \leq 4$
Case 3	Late-harvest of X1 Early-harvest of X2	Weibull (0, β, 1.5)	Weibull (0, β, 0.5) $1 \leq \beta \leq 10$
Case 4	Late-harvest of X1 X2:early-harvest→late-harvest	Weibull (0, 1, 2.2)	Weibull (0, 1, γ_2), $0.3 \leq \gamma_2 \leq 5$

where $\beta_2 = 1+ k$, k = 0.1, 0.2,…, 6, where k is the distance between β_1 and β_2 and is denoted as the scale effect. X_1 has a standard Weibull distribution in Cases 1 and 2. Cases 1, 2, and 3 investigate, for different time-varying situations, the scale effect of the DS payoff on the NE payoff distribution. Case 4 investigates how a change in γ affects the NE payoff distribution. Our model has two random variables which interact with each other. The players know that the decision rule of each player is $Y = MAX(X_1, X_2)$, which is the probability distribution transformation of X_1 and X_2.

2.1. Simulation Procedures

We used a desktop running the Windows 7 system to run a C++ program, that is, the probability distribution simulator. Here, we chose the Weibull distribution, X ~ Weibull (α, β, γ), and its algorithm equation $X = \alpha + \beta\ (-1 \times log(\mathrm{RND}))^{1/\gamma}$ to generate the values of strategic payoffs on the condition of different parameters, and then transferred to the values of the NE payoffs.[3] The simulation steps of Case 1 are as follows.

[3]The software of the probability distribution simulator is form C.C.C. Ltd. (http://psccc.com.tw/en)

Step 1. Set 61 random variables following the Weibull distribution with $1 \leq \beta \leq 61$ and $\gamma_1 = \gamma_2 = 2$. X_1 has the condition $\beta = \beta_1 = 1$, and X_2 has the condition $1.1 \leq \beta_2 \leq 61$.

Step 2. Simulate the values from the Weibull distribution for X_1 and X_2.

Step 3. Choose $X_1$and X_2 with different β_2 to perform MAX(X_1, X_2) and then calculate the probability distribution of the NE payoff, Y = MAX(X_1, X_2).

Following the above steps, we also simulated Case 2 with the conditions $\beta_1 = 1$, $\gamma_1 = \gamma_2 = 0.2$, and $0.1 \leq \beta_2 – \beta_1 \leq 3$, and Case 3 with the conditions $\gamma_1 = 1.5$, $\gamma_2 = 0.5$, and $1 \leq \beta_2 = \beta_1 \leq 10$, in order to see how different β_2 values change the NE payoffs. The simulation steps of Case 4 are as follows.

Step 1. Set 49 random variables following the Weibull distribution with $\beta_1 = \beta_2 = 1$. X_1 has the condition of $\gamma_1 = 2.2$, and X_2 has the condition of $0.3 \leq \gamma_2 \leq 5$.

Step 2. Simulate the values from the Weibull distribution.

Step 3. Choose $X_1$and X_2 with different γ_1 to perform MAX(X_1, X_2) and then calculate the probability distribution of the Nash equilibrium, Y=MAX(X_1, X_2).

In fact, this methodology is a type of big-data analysis. At the second step, each random variable generates 60 million datapoints, and Y is generated from 120 million datapoints and refined as 60,000 million datapoints.

3. Results

3.1. The scale effect

To explain the changing effect of the scale parameter on the NE payoff distributions, first, notice that the NE payoff distribution shows the probability of the realized payoff when players know the game structure and the density function from the decision rule, MAX(X_1, X_2). For a detailed interpretation, let us focus on 0 which shows two examples of the NE payoff distributions with a change in the scale parameter in a late-harvest situation. The values of k dominate the NE payoffs such that the means, variances, and skewness and kurtosis coefficients are different from X_1 and X_2.

0 shows that the higher the value of $\beta_{2,}$ the higher the average, risk, and skewness and kurtosis coefficients. Here, we can find that the risks of strategic payoffs induce the existence of a risk premium so that the average of the NE payoff is higher than the average of the DS payoff. We also find that the shapes of the distributions in Table 2 are not the same as the X_2 distributions. Next, if β_2 increases, how do the NE payoff distributions generate the patterns of coefficients?

Table 2. The NE payoff distributions in Case 1

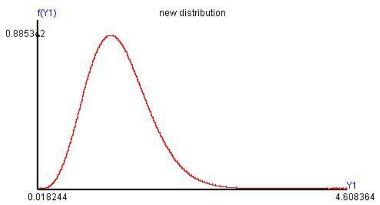
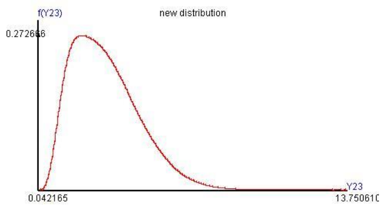
Y1=MAX(X1, X2(0,2,2))		Y23=MAX(X1, X24(0,3.3,2))	
			
Mathematical Mean:	1.20534	Mathematical Mean:	2.96244
Geometrical Mean :	1.11394	Geometrical Mean :	2.58358
Harmonic Mean :	1.01179	Harmonic Mean :	2.18560
Variance :	0.20969	Variance :	2.19682
S.D. :	0.45792	S.D. :	1.48217
Skewed Coef. :	0.52945	Skewed Coef. :	0.73966
Kurtosis Coef. :	3.30351	Kurtosis Coef. :	3.36729

Figure 1 shows, for given late-harvest strategies, how the coefficients of the NE payoff distributions are patterned when the values of k go from 0.1 to 6. First, both the means and variances have a positive relation with k, but the means have a linear relation with k, while the variances are convex with respect to k. Intuitionally speaking, when players have late-harvest strategies, the higher the scale parameter of the DS payoff, the higher the average of the NE payoff that is dominated by the DS payoff. Considering the uncertainty in decision-making, the scale effect indicates that that the risk of the NE payoff increases faster than the average of the NE payoff. We also show that the NE payoff distributions are positively-skewed, and, in particular, the highest skewness coefficient occurs at k = 1.

The pattern of skewness coefficients initially increases and then decreases to 0.65 when k is from 0.1 to 6. The kurtosis coefficients have the same pattern as the skewness coefficients, but the highest value of the kurtosis coefficient occurs at k = 0.7, after which the kurtosis coefficients tend to stabilize around 3.25. Thus, we obtain Result 1 as follows.

Result 1. *In the decision-making of two late-harvest strategies, we find that*

(1) *dE(NE payoff) / dk> 0, dVar(NE payoff) / dk> 0*

(2) $d^2E(NE\ payoff) / dk^2 = 0$, $d^2Var(NE\ payoff) / dk^2 > 0$.

(3) *The skewness coefficient > 0 and the kurtosis coefficient > 3.2.*

(4) *When k ≤ 0.7, d skewness / dk> 0 and d kurtosis / dk> 0.*

(5) *When k > 0.7, d kurtosis / dk< d skewness / dk< 0.*

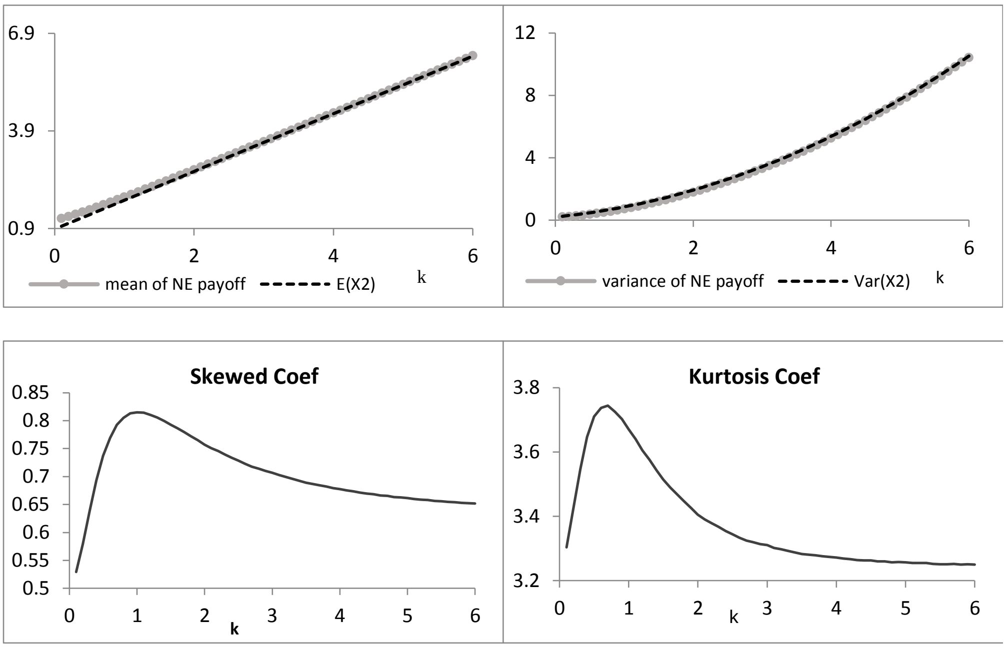


Figure 1. The coefficient patterns of the NE payoffs with the changed values of k in Case 1

Figure 1 also shows that when players face two late-harvest strategies and payoff uncertainty, the NE payoff distribution is changed by the scale parameter of the DS payoff. Compared with the DS payoff, X_2, one interesting point is at k = 0.1 which has the highest average and the smallest risk of the NE payoff. When k > 0.1, the higher k reduces the difference between the averages of the NE payoff and the DS payoff, E(NE payoff) – E(X_2), but increases the difference between variances of the NE payoff and the DS payoff, Var(NE payoff) – Var(X_2), until k = 1.5, after which the difference becomes smaller. We obtain Result 2 as follows.

Result 2. *Comparison of the NE payoff and the DS payoff in Case 1,*

(1) $d(E(NE\ payoff) – E(X_2)) / dk < 0$.

(2) *When* $0.1 \le k \le 1.5$, $d(Var(NE\ payoff) – Var(X_2)) / dk > 0$.

(3) *When* $1.5 < k \le 6$, $d(Var(NE\ payoff) – Var(X_2)) / dk < 0$.

The first row in Figure 2 illustrates the patterns of E(NE payoff) and Var(NE payoff) in Case 2 while the second row in Figure 2 explores the patterns of E(NE payoff) and Var(NE payoff) in Case 3. Figure 2 shows the linear relation between E(NE payoff) and k in Cases 2 and 3. In other words, the higher k leads to more risk and a stable increasing mean in equilibrium. Figure 2 also shows the convex relation between Var(NE payoff) and k in Case 2 and Case 3. However, Var(NE payoff) fluctuates in Case 2 than in Case 3. The difference between Var(NE payoff) and Var(X_2) reveals a similar trend of risk premium in Case 2. Thus, this reveals that E(NE payoff) and Var(NE payoff) are dominated by the DS payoff distribution, so that there are similar fluctuations of risk premium and Var(NE payoff) – Var(X_2) after decision-making. In other words, the risk premium can represent the risk of strategic payoffs in our model setting where players do not know the realized strategic payoffs, and they have to bear the risk of decision.

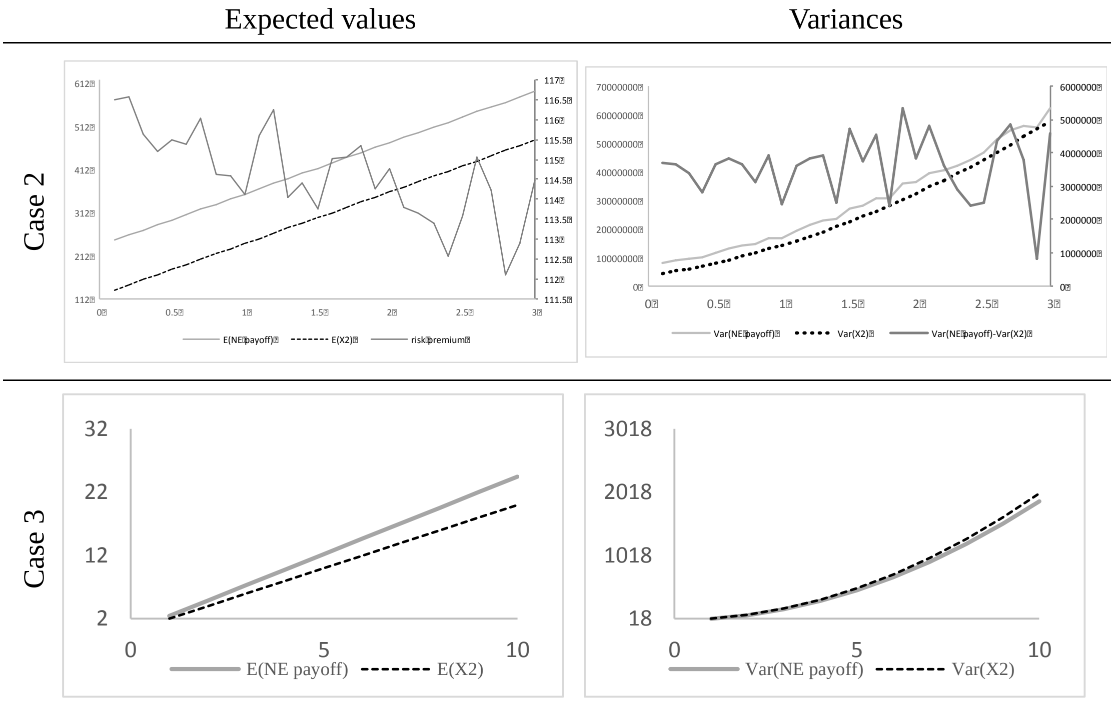


Figure 2. The coefficient patterns of the NE payoffs with the changed values of k in Case 2 and 3

Case 3 in Figure 2 shows that the higher proportion of scale parameter in two strategic payoffs causes the NE payoff has higher average of the NE payoff and lower risk of the NE payoff. The outcomes in Case 3 are different from Case 1 and 2. The reason is that E(NE payoff) is dominated by the DS payoff, but at the same time, the NDS payoff has the scale effect and time-varying effect on the NE payoff to decrease the average and risk. Besides, the same proportion of scale parameter leads to a main interaction effect from time-varying effect on the NE payoff. Thus, we can obtain the result 3.

Result 3.

(1) *Whatever players face the early-harvest or late-harvest strategies, the average of NE payoff increases with k, meanwhile, corresponding to higher convex risk.*

(2) *The NE payoff with higher mean and lower risk induces from two time-varying strategies and the same proportion scale parameter.*

Comparing with Case 1, 2 and 3, we show that the two early-harvest strategies lead to the highest average and risk of the NE payoff. First, the decision between two early-harvest strategies has the largest average and risk of the NE payoff in Case 2, meanwhile, the smallest average and risk of the NE payoff occur in the decision between two late-harvest strategies in Case 1.Second, the higher the k is, the higher the average of the NE payoff is and the lower the risk of the NE payoff is in Case 1, and so does in Case 3. More specially, Case 2 has the higher average and risk of the NE payoff. The property of the NDS, early-harvest or late-harvest, has important role in the decision of the NE payoff when the DS is early-harvest strategy.

Third, the k and the DS payoff can efficiently help players to forecast the average of NE payoff whatever they use early-harvest or late-harvest strategies, however, it is not easy to forecast the risk of the NE payoff, such as Case 2. If the two strategic payoffs have the changed scale parameter simultaneously, then the average of the NE payoff can be forecasted by the k and the DS payoff.

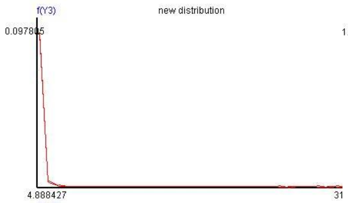
Finally, we show that the decision of the late-harvest strategies leads to the closest outcomes between the NE payoff and the DS payoff. This reveals that as time goes by, the NE payoffs will the same as the DS payoffs by the large k. This also reveals that in the long run the NE payoff is the DS payoff when players use the strategies with large enough means of strategic payoffs.

3.2. The time-varying effect

0 illustrates two examples of early-harvest and late-harvest DS when X_1 is a late-harvest strategy. For a given late-harvest strategy of X_1, all the coefficients of the NE payoffs decrease when X_2 changes from early-harvest to late-harvest. According to the payoff uncertainty, skewness and kurtosis coefficients indicate that the higher the value of γ, the less skewed and centralized the NE payoff. In particular, the right graph has the highest probability of the NE payoff and a smaller range and

values of the NE payoff. Thus, the shape parameter has an important effect on the NE payoff distribution.

Table 3. The NE payoff distributions in Case 4

<table>
<tr><th>Y=MAX(X1, X2(0,1,0.5))</th><th>Y=MAX(X1, X2(0,1,5))</th><th>Y=MAX(X1, X2(0,1,15))</th></tr>
<tr>
<td>
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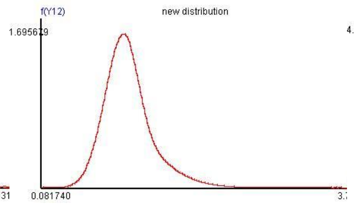
<td>
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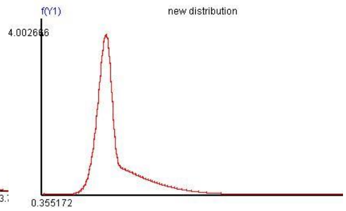
<td>
</td>
</tr>
<tr>
<td>Mathematical Mean: 2.41928
Geometrical Mean :1.36212
Harmonic Mean :0.93700
Variance :18.71743
S.D. : 4.32636
Skewed Coef. : 7.09920
Kurtosis Coef. : 99.60394</td>
<td>Mathematical Mean: 1.09356
Geometrical Mean :1.05698
Harmonic Mean :1.02089
Variance :0.08453
S.D. : 0.29074
Skewed Coef. : 0.94221
Kurtosis Coef. : 4.87492</td>
<td>Mathematical Mean: 1.10527
Geometrical Mean :1.08212
Geometrical Mean :1.06284
Variance : 0.06239
S.D. : 0.24979
Skewed Coef. : 1.90890
Kurtosis Coef. : 7.35377</td>
</tr>
</table>

Figure 3 illustrates the coefficient patterns of the NE payoff when the NDS is late-harvest and the DS is from early-harvest to late-harvest. Due to the bathtub curve and the property of $E(X_2)$, the range of γ_2 is divided into three parts, [0.3, 0.9], [1, 2.2], and [2.3, 4]. When γ_2 is in [0.3, 0.9], the DS is early-harvest. In this situation, with the higher time-varying effect of the early-harvest DS, the coefficients in Figure 3 rapidly and evidently decreases with the increase of γ_2, especially at the beginning. This means that the patterns of E(NE payoff) and Var(NE payoff) induced from the early-harvest DS are close to the late-harvest NDS in the decision-making process. The decline of patterns in the bottom row also reveals the time-varying effect on the skewness and kurtosis coefficients.

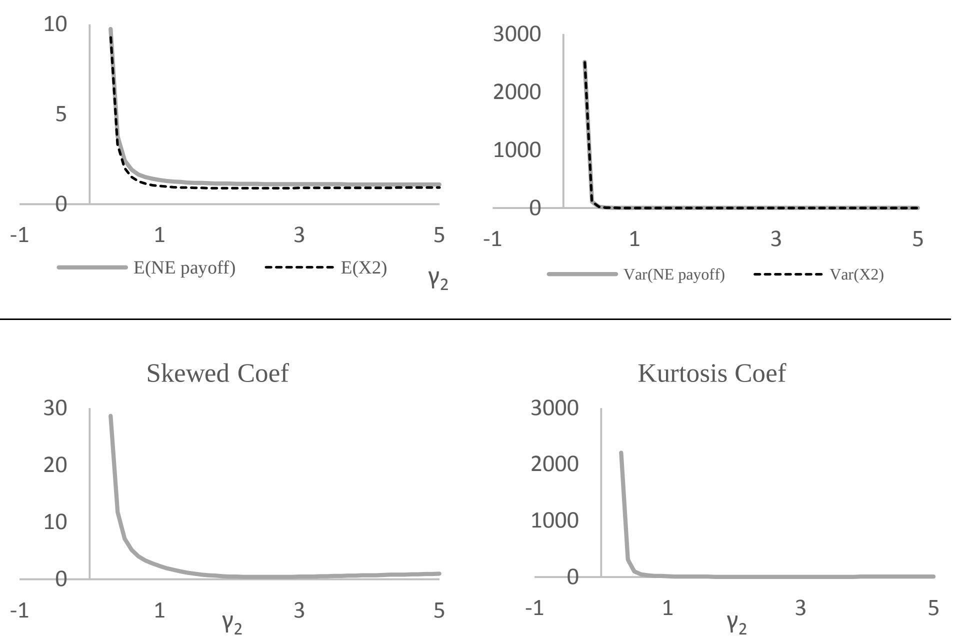


Figure 3. The patterns of coefficients of the NE payoff distributionin Case 4

In the range of [1, 2.2], E(NE payoff), Var(NE payoff) and the kurtosis coefficient fall slightly and become stable, but the skewness coefficient decreases gradually. The slow down speeds of coefficients indicate the long-term effect on the decision-making. In the range of [2.3, 4], there is a mostly stable trend of E(NE payoff) and Var(NE payoff). However, the skewness coefficient achieved the smallest value, 0.38375, at $\gamma_2 = 2.5$, and then slowly increased. The kurtosis coefficient, similarly to the skewness coefficient, achieved the smallest value, 3.10383, at $\gamma_2 = 2.3$, and then increased. This highlights that if the time is long enough, the NE payoff will become more skewed and centralized, while the risk of the NE payoff is as small as possible. Thus, we obtain Result 4 below.

Result 4. *When players face the DS from early-harvest to late-harvest, all the coefficients of the NE payoff decrease and then become stable. The properties* *include:*

(1) *E(NE payoff) – E(X_2) > 0, ΔE(NE payoff) < 0 andΔE(X_2) < 0for all γ_2.*

(2) *Var(NE payoff) – Var(X_2) < 0 if $0.4 \le \gamma_2 \le 2.7$.*

(3) *The pattern of skewness is a hyperbolic form where the smallest value is 0.38375 at $\gamma_2 = 2.5$.*

(4) *The pattern of centralization is a quadratic form where the smallest value is 3.10383 at $\gamma_2 = 2.3$.*

For a given the NDS of late-harvest, the time-varying effect of the DS payoff plays an important role in the decision process. When the DS payoff is from early-harvest to late-harvest, the long-term effect gradually becomes more significant in the decision process so that the NE payoff has a lower risk and return. That is the evidence of the capital asset pricing model (CAPM) due to the property of the Weibull distribution and the assumption of the DS and the other late-harvest strategy. Thus, the later the strategic payoff is realized, the lower the NE risk faced. Second, the shape of E(X_2) with respect to γ_2 is a hyperbolic form with properties that rapidly decrease at $\gamma_2 < 2.2$, and slowly increase at $\gamma_2 > 2.2$. The skewness and kurtosis coefficients imply that less skewness and centralization occurs when the shape parameter of the DS is close to 2.2, but more skewness and centralization occurs when the shape parameter of the DS is larger than 2.2.

3.3. Comparison of E(NE payoff) and risk

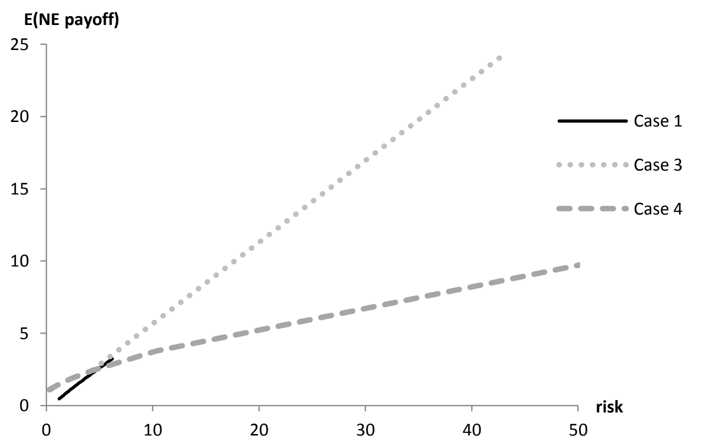
Figure 4 illustrates the relationship between E(NE payoff) and sd(NE payoff) of Case 1, 3 and 4, where sd(.) represents standard deviation. The horizontal axis is the value of sd(NE payoff) and the vertical axis is the value of E(NE payoff). Each line represents a market line of CAPM. We obtain Result 5 as follows.

Result 5.

(1) *The time-varying effect and the scale effect play important roles in the market lines of the NE payoff.*

(2) *Whatever the time-varying effect and the scale effect are, the higher the risk, the higher the expected value of the NE payoff; that is, ΔE(NE payoff) / Δsd(NE payoff) > 0.*

More specifically, in Case 4, the larger expected values are from the early-harvest DS. In particular, the average slope is 0.286 when γ_2 is in [0.3, 0.9], the average slope is 0.442 when γ_2 is in [1, 2.9], and the average slope is 0.190 when γ_2 is in [3, 5]. We can also calculate the average slope as -0.285 when γ_2 is from 5 to 15. The largest slope is 0.475 when γ_2 is from 1.6 to 1.7.



x is standard deviation and y is expected value in the estimated equations.

Figure 4. The relationships between E(NE payoff) and sd (NE payoff)

We also show the coefficients of estimated equations in each case shown in Table 4. The coefficient of the risk is represented as a time-varying effect. When the time is long term, the time-varying effect eliminates the payoff risk, and so the risk does not induce a larger effect on the NE payoff.

Thus, for the NE payoff in Case 4, the higher the risk is, the higher the expected value is, which contrasts with Case 1. In Case 2, for given early-harvest strategies, the scale effect has the response in the intercept, while the additional 0.00678 units of E(NE payoff) correspond to an additional one unit of risk. However, Case 1 seriously weakens the scale effect. Case 3 with one early-harvest DS and the late-harvest NDS is heavily affected by the risk, as shown by the slope, 1.7697.

Table 4. The coefficients of estimated market lines in each case

	Case 1	Case 2	Case 3	Case 4
Intercept	-0.1749	64.914	0.0021	1.1096
Coefficient	0.5524	0.00678	1.7697	0.1767
R^2	0.99952	0.99713	1	0.98229

According to the situations with NE payoffs having a different relationship between the expected value and the risk, the time-varying effect leads to different results in Case 1, 2, and 3, even though the scale effect increases. For instance, Case 1 and Case 2 represent the late-harvest and early-harvest situations with the change inthe scale effect. The time-varying effect from Case 2 to Case 1 weakens the intercept but strengthens the slope, which is a result of the change in risk. This indicates that the time-varying effect can weaken the risk.

The time-varying effect and scale effect have different influences on the market lines as shown in Case 1 and Case 4 by maintaining the NDS's parameters but different forms changed the parameters of the DS payoff. For a given value of risk, an additional one unit of risk leads to an increase of 0.5524 units of expected value in Case 1 and 0.1767 units of expected value in Case 4. That is, the lower expected value is induced from the time-varying effect. Figure 4 shows that for any value of the risk in [0, 3.41895], the expected value in Case 4 is larger than in Case 1, and the opposite holds in (3.41895, ∞).

Therefore, it is worthy to note which situations the strategic payoff distributions are in, as players face different relationships between the expected value and risk in different situations. Thus, we obtain Result 6 as follows.

Result 6.

(1) *The slopes are sorted as follows: Case 3 > Case 1 > Case 4 > Case 2.*

(2) *The intercepts are sorted as follows: Case 2 > Case 4 > Case 3 > Case 1.*

(3) *The expected values of the NE payoffs have the following relationships:*

(a) *Case 2 > Case 1 at [0, 119.29346],*

(b) *Case 4 > Case 1 at [0, 3.41895],*

(c) *Case 2 > Case 3 at [0, 36.82067],*

(d) *Case 2 > Case 4 at [0, 375.49670], and*

(e) *Case 4 > Case 3 at [0, 0.69523].*

Result 6 shows that players face the market lines with different time-varying effect and scale effect, and know better expected values of the NE payoff with the different situations in a specific risk level. We show that the pure late-harvest situation with the scale effect has a large enough risk to increase the time-varying effect, that is, Result 6-3-a. The pure time-varying effect situation (Case 4) can have higher expected values in comparison with Case 1 for small risks, but with Case 2 for large risks, we apply Results 6-3-b and 6-3-d. In other words, the higher the risk, the greater the domination of the pure time-varying effect on the early-harvest situation with the scale effect.

4. Conclusion

Decision-making analysis is a set of mechanisms supposed to resolve optimal strategies problems in decision and to provide incentives to players such as investors, firms and anyone who interact with other persons. This is surprising as players have detailed knowledge about the strategic payoff uncertainty represented by the probability distribution from historical information and can hence be a crucial source of information for the decision-making about time-varying and payoff-scaling.

Our model has the advantages of assuming Weibull distribution on strategic payoffs are as follows. First, the bathtub curve of Weibull distribution can help to explain time-varying effect from the early harvest to late harvest by the range of shape parameter. This can complement to the existing literature on the game theory with time-varying effect and uncertainty. Second, the change of shape parameter can generate the trend of the NE payoffs from early-harvest to late-harvest situation. Third, for a given shape parameter and the scale parameter of the NDSpayoff, the NE payoffs can be patterned by the scale parameters of the DS payoff. Finally, the pattern of the NE payoffs can be compared with situations with different scale parameter and shape parameter.

As intuition would have it, the key parameter affecting the NE payoffs is the interaction between two uncertain strategic payoffs, as a higher scale parameter of the DS payoff strengths the average and weakens the risk of the NE payoffs. This also guarantees the capital asset pricing model (CAPM) is a good model for explanation of uncertainty in game theory. Thus, we qualify these insights by endogenizing the two uncertain strategic payoffs with Weibull distribution. In doing so, we can contribute to the literature on 2 × 2 game, which emphasizes the uncertainty and time-varying of strategic payoffs.

REFERENCES

[1] **Cassidy, R.G., Field, C.A.and Kirby, M.J.L. (1972),** ***Solution of a Satisfying Model for Random Payoff Games***. *Management Science*, 19, 266-271;

[2] **Harsanyi, J.C. (1973),** ***Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points***. *International Journal of Game Theory*, 2, 1-23;

[3] **Lee, M.Y. (2014),** ***Strategic Payoffs of Normal Distribution Bump into Nash Equilibrium in 2 × 2 Game***. *International Journal of Game Theory and Technology*, 2, 1-10;

[4] **Lee, Y.H. and Lee, M.Y. (2014),** ***Risky Strategies with Payoff Mean Changed in 2 × 2 Simulation-Based Game: A Normal Distribution Case*****.** *Problems and Perspectives in Management*. 12, 503-511;

[5] **Lee, Y.H. and Lee, M.Y.(2015),** ***The Payoff Pattern of Nash Equilibra by a Change of Risk in 2 × 2 Simulation-Based Game***. *Frontiers in Artificial Intelligence and Applications*, 274, 2143-2151;

[6] **Varian, H.R. (2009),** ***Intermediate Microeconomics: A Modern Approach*****;** W. W. Norton and Company, New York;

[7] **Vorobeychik, Y. and Wellman, M.P. (2008),** ***Stochastic Search Methods for Nash Equilibrium Approximation in Simulation-Based Games***. Seventh International Joint Conference on Autonomous Agents and Multiagent Systems, 1055-1062;

[8] **Vorobeychilk, Y. (2010),** ***Probabilistic Analysis of Simulation-Based Games***. *ACM Transition Modeling and Computer Simulation*, 20, Article No. 16.